Available online at www.sciencedirect.com
SCIENCE DIRECT®
ELSEVIER
International Journal of Solids and Structures 42 (2005) 4514–4515
INTERNATIONAL JOURNAL OF SOLIDS and STRUCTURES
www.elsevier.com/locate/ijsolstr

Discussion

Discussion on "Exact expansions of arbitrary tensor functions F(A) and their derivatives"

Guansuo Dui *

Institute of Engineering Mechanics, Beijing Jiaotong University, Beijing, 100044, PR China

Received 2 April 2004; received in revised form 1 September 2004
Available online 24 February 2005

In a recent paper Lu (2004) presented a general method for determining the exact expansions of a tensor function $\mathbf{F}(\mathbf{A})$ in n-dimensional Euclidean space. Using this method, closed-form, singularity-free expressions of arbitrary tensor functions and their first derivatives are deduced in two- and three-dimensional cases. However, the work raises several issues, some of which we discuss in the following points:

1. The tensor function dealt with in the paper is only a very special case, since an arbitrary tensor function cannot be expressed as a power series form. Even for a very simple tensor function

$$\mathbf{F}(\mathbf{A}) = \mathbf{I}\ \mathrm{tr}\ \mathbf{A} \tag{1}$$

we are also difficult to find the power series expression. But the derivatives of (1) can be calculated easily, such that

$$\frac{\mathbf{dF}(\mathbf{A})}{\mathbf{DA}} = \mathbf{I} \otimes \mathbf{I} \tag{2}$$

2. It is difficult to discern the necessary of the generating function $\mathbf{G}(\mathbf{A})$ introduced in the paper.
   (1) $\mathbf{G}(\mathbf{A})$ does not appear in the final results.
   (2) In some cases, the expression of $\mathbf{G}(\mathbf{A})$ does not exist the close form, for example

$$\mathbf{F}(\mathbf{A}) = \mathbf{A}^{-1} \sin \mathbf{A} \tag{3}$$

The generating function $g(x) = \int x^{-1} \sin x \, \mathrm{d}x$ is not easy to be found.

3. Taking the derivative on both sides of (62) and using the chain rule to find $\frac{\mathbf{dF}(\mathbf{A})}{\mathbf{DA}}$ is the main point in the paper. However, there is a serious problem with this approach. We should be aware of that the coefficients in (62) may not be differentiable even if $\mathbf{F}(\mathbf{A})$ is differentiable (Ball, 1984; Man, 1994). Hence the formula (72) must be regarded as incomplete ones.

* Tel.: +86 1051688417; fax: +86 1051682094.
E-mail address: gsdui@center.njtu.edu.cn

0020-7683/$ - see front matter
doi:10.1016/j.ijsolstr.2004.12.020

4. For the case when $\mathbf{A}$ has three distinct eigenvalues, we can easily find a simple representation. Since $\mathbf{A}$ has the spectral decomposition (Balendran and NematNasser, 1996)

$$\mathbf{A} = \sum_i \lambda_i \mathbf{N}_i \otimes \widehat{\mathbf{N}}_i. \tag{4}$$

Following the approach by Dui et al. (1999) results in

$$\frac{\mathrm{d}\mathbf{F}(\mathbf{A})}{\mathrm{d}\mathbf{A}} = [(\mathbf{F}'(\mathbf{A}) - \varphi_1\mathbf{I} - 2\varphi_2\mathbf{A})\boxtimes\mathbf{I}] : \left[\mathbf{I}\boxtimes\mathbf{I} - \Delta_-^{-2}\left(\hat{\mathbf{A}}\boxtimes\hat{\mathbf{A}}^{\mathrm{T}}\right) : \left(\mathbf{A}^2\boxtimes\mathbf{I} - 2\mathbf{A}\boxtimes\mathbf{A}^{\mathrm{T}} + \mathbf{I}\boxtimes\left(\mathbf{A}^{\mathrm{T}}\right)^2\right)\right] + \varphi_1\mathbf{I}\boxtimes\mathbf{I} + \varphi_2\left(\mathbf{A}\boxtimes\mathbf{I} + \mathbf{I}\boxtimes\mathbf{A}^{\mathrm{T}}\right) \tag{5}$$

where

$$\widehat{\mathbf{A}} = (\mathbf{I}_1^2 - 4I_2)\mathbf{I} + 2I_1\mathbf{A} - 3\mathbf{A}^2, \quad \Delta_-^2 = \det\widehat{\mathbf{A}}. \tag{6}$$

Clearly, we do not need to determine complicated coefficients in (5).

5. It should be also noted that some main final results in the paper have been appeared in the early work (Balendran and NematNasser, 1996) when $\mathbf{A}$ has repeated eigenvalues. For example, the formulae (67), (77) and (78) were derived by Balendran and NematNasser (1996) in (3.2a–e) (4.12b) and (4.12c), respectively.

In spite of that, the purpose of yielding an explicit tensor representation of the tensor function when the $\mathbf{A}$ is not symmetric in n-dimensional space is an interesting and valuable addition to the present state.

Acknowledgement

The author would like to express his gratitude to the Science Foundation of NJTU (Grant No. PD-247) for support to this work.

References

Ball, J.M., 1984. Differentiability properties of symmetric and isotropic functions. Duke Math. J. 51, 699–728.

Balendran, B., NematNasser, S., 1996. Derivative of a function of a nonsymmetric second-order tensor. Q. Appl. Math. 54, 583–600.

Dui, G., Ren, Q., Shen, Z., 1999. Time rates of Hill's strain tensors. J. Elasticity 54, 129–140.

Lu, J., 2004. Exact expansions of arbitrary tensor functions $\mathbf{F}(\mathbf{A})$ and their derivatives. Int. J. Solids Struct. 41, 337–349.

Man, C.-S., 1994. Remarks on the continuity of the scalar coefficients in the representation $\mathbf{H}(\mathbf{A}) = \alpha\mathbf{I} + \beta\mathbf{A} + \gamma\mathbf{A}^2$ for isotropic tensor functions. J. Elasticity 34, 229–238.